A fruitful start

If I were to start by saying that I had a few brief and preliminary points to make, your experience with such indefinite terms as "few" and "brief" might just cause you a little uneasiness.

When I say instead that I'm going to introduce two fields of inquiry and then combine them in one example, you probably sense that I know where I'm going and won't waste your time. Using even the two simplest numbers can make that difference.

The first field I have in mind is math. Call it numbers, figures, mathematics, abstract relations, or what you will. The second field is semantics. Think of it as the full range of meanings of any language or symbol-system, including math, working away inside us.

My own semantic interest in numbers stems from parents who spoke different number languages, a CPA father and a chanteuse mother.

I can still remember Mother murmuring her sums, usually bridge scores or prices scribbled on brown grocery bags. "Trente, trente et soixante font quatre-vingt-dix, quatre-vingt-dix et cent font. . . ." Most everything else except singing she did in English, but she said she never learned to add or subtract except aloud and in French. Dad, on the contrary, added only silently, very fast.
I also remember Mother’s beautiful numerals, tall, thin, gracefully curved; her one with the long, almost vanishing, upswing preceding its more definite downswing, her seven with its honestly European crossbar, and her nine gently curving down below the line. Compared with these beauties, Dad’s numerals were just workaday dwarfs, cramped, not more than one-fourth the height or breadth of Mother’s, and individually jagged even though lined up in precise rows.

Dad had exact ideas about numbers where Mother tended to round them out. I can remember her saying once that he was difficult when it came to money. If she owed him $1.02, she said, he insisted on getting the last two pennies; he wouldn’t just take the dollar; and if he owed her 97 cents, he insisted on getting his three cents change. No wonder I learned, in the practical way children do, that when I needed to borrow money, ask Mother, but when I needed someone to hold funds for me, ask Dad.

My adult interest in numbers sprang from a career in market research and peaked sharply during two hundred cross-examinations under oath as an expert witness defending aviation market-research figures developed by my staff.

To survive the witness stand I needed to avoid error. To this end I devised in 1969 a recruitment quiz to test how well applicants avoided the kinds of numerical mistakes, mostly semantic in origin, I had by then made and noticed.

What I’ll tell you in this book derives mainly from my own errors and the answers given by the one hundred ninety-six clerical applicants from 1969 through 1984 who had sounded the most promising of those answering our ads containing the qualifier “Good at numbers.”

Apart from linguistic specialists, most people’s acquaintance with semantics stems directly or indirectly from the general semantics of Alfred Korzybski (1879–1950), which enjoyed its greatest vogue in the 1940s. I remember Dad talking about Stuart Chase’s 1938 popularization, *The Tyranny of Words*, when I would have been about thirteen, so I heard early on that the very language we speak might have some strange power to rule us.
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General semantics certainly had then and still has profound implications. Many of these implications, however, were diluted as general semantics entered our culture's main stream. For example, Korzybski's dictum, "the word is not the thing," seems to have become "it's only a semantic problem," rather than the deeper puzzler, "whatever you say it is, it isn't."

As a practicing general semanticist, I have often derived a business advantage from semantic principles and their overlooked implications. Take the principle that cow, is not cow,. No real cow is the same as the word "cow" nor is any real cow the same as any other real cow. Thing, is not the same as thing,. Only our calling them by the same word makes it seem so.

It then follows that whenever we add two things together we add two different things. Any real-world addition, so to speak, adds apples and oranges.

If my third-grade teacher, Miss Chapman, was literally right that "we can't add apples and oranges," then it follows that only numbers can be added, not things. That's logical but impractical and operationally false. We do demonstrably add all sorts of "things" together. So what Miss Chapman must have meant was that the things we add must bear the same name. If we call cow, a "cow" and cow, a "cow," then we can have two cows. If we call fruit, a "fruit" and fruit, a "fruit," then we can add them even if fruit, is an "apple" and fruit, is an "orange." But Miss Chapman didn't explain this.

Business data depend on such naming abilities. To obtain company size we add clerks and managers together as "employees." To get a machine's full cost we add its price to its maintenance and other factors. In my business I regularly add airline scheduling, air fares, and airport accessibility together to obtain the datum we call a "service-value index," a useful number reflecting the convenience of making a round trip in any given city-pair market.

To discover which applicants had this number-related naming ability, the third question in my quiz asked applicants to solve the following combined-math-and-semantics problem in addition:
The one hundred ninety-six applicants answering this question gave fifty-six digitally different answers. I say “digitally,” because if penmanship mattered, they gave one hundred ninety-six different answers. Now, if I had to honor the uniqueness of each answer, I’d not be able to discuss them with you in an intelligible manner. So for your convenience I’ve added the apples and oranges of the answers together in seven groupings. I trust I have your blessing in this.

The most frequent answer was the one I’d hoped for: “7 fruit.” That includes such answers as “seven fruits,” “seven pieces of fruit,” and other variants. Fifty-two applicants gave this answer, or just over one in four of all applicants.

The next most frequent answer was “7.” Just seven, not seven of anything. Forty-six gave this answer, or just under one in four of all applicants.

The next most frequent answer was “2 apples + 5 oranges.” I must confess I fail to see how this solves any problem in addition. Thirty-six applicants, or just over one in six, took this tack.

More forthright, perhaps, were the applicants who gave the answers I group under the heading, “can’t add unlike values.” I put twenty-seven applicants, or more than one in eight, in this group, twenty-one because they answered surrounding problems but skipped this one, and six because their answers explicitly rejected the addition of unlikes.

A still less popular answer was “7 apples and oranges.” That’s not bad. But without hyphens to make “apples-and-oranges” a new unit—and they were never used—I don’t see it as solving the naming problem in addition. Fifteen gave this answer, or about one applicant in thirteen.

Another fifteen applicants waffled. One wrote, “7 or 2 apple 5 oranges,” which misses a plural, errs on both sides of the straddle, and equivocates. You might say I was lenient to count such a reply as only one error.
Finally, five applicants, or about one in forty, gave clear-cut wrong answers like "7 oranges." I doubt even today's genetic engineers could make two apples and five oranges into seven oranges.

Note that the answers resemble the party vote in some European elections. Each attracts a minority. The most frequent answer, "7 fruit," is the winner. The other answers don't solve the naming problem. Mostly they avoid it, by dropping all mention of units, by repeating "apples and oranges," by declaring a solution to be impossible, or by equivocating.

You can imagine how applicants protested this question. Lucky you. My wife, who's our personnel director, and I had to face the protests. I tend to side with the underdog, so I heard out the claims of unfairness and trickery. I agree that the protesters were duped. They were duped by a rule, "you can't add apples and oranges," that glosses over a very real semantic problem.

Outside math class we never add anything except "apples" and "oranges," so—and here's the semantic problem—how do we do it? No applicant I interviewed recalled having been told how. All recognized the "apples-oranges" problem, but none had developed a general solution. Indeed, none seemed ever to have addressed the semantic question, "If no two things are alike, what are the rules for adding things?"

Many applicants said they could have solved the problem had they known what answer we wanted. Think about that for a moment. It's either a particularly simple-minded statement or they're trying to tell us that they're willing to accept whatever the house rule is. Forget math and semantics.

Some applicants insisted the problem truly was not solvable. I'd then put my finger to my lips to signal a Korzybskian, let's-get-beyond-words silence, and, pointing to a red pencil and a green pencil, quietly say, "Please add these." Back came the answer, "Two pencils." "But," I lied, "in this office we call this [pointing to the green pencil] a 'greenie' and this [pointing to the red] a 'reddie.' You can't add greenies and reddies." "Oh yes I can," came the reply, "because they're both pencils."

"Oh," said I, adding a pen to the collection, "then what about
these?" "Three writing implements," I was now told without hesitation. A stapler made it "four desk implements," and my wastepaper basket made it "five office objects."

"So," I chortled, "whether you can add things together depends on what they're called, is that right?" This evoked only puzzled looks, so I went on, "If I call them 'fruit,' you can add them together, but if I call them 'apples' and 'oranges,' you can't. That makes it a question of who will do the naming, doesn't it?"

Some still insisted that you could never add apples and oranges no matter what they were called. I'd like to think of that not as a considered opinion but as a payback for putting applicants on the spot. Okay, I'm at fault there. But I fear a few really believed the addition was impossible. And almost nobody wanted to think about it. That seems to be part of the price we pay for divorcing arithmetic and semantics. The divorce gulls us into treating math problems as "just math problems" and semantic problems as "just semantic problems."

Anyone who accepts that you can't add apples and oranges attempts the near impossible, to live without adding. It would never work in business. The accounting department has to add apples, oranges, and pomegranates every day. How can they? Easy. They just call them assets and express them in dollars. Sales, engineering, production, personnel; they all do that sort of thing, don't they?

The question is not whether we can add different things, but how we can add them in clear and useful ways. That gets us into meanings, into semantics, with both feet.

By looking at math and semantic aspects together, we can develop a better feel for each. The logical purity of mathematical processes such as addition then shines forth brightly. So does the semantic risk of applying the process to the world. We thus reach a fact-of-life truth: that all worldly additions, all extensional additions, combine different things, and there are no final rules on how to do it.

One useful but incomplete rule for combining different things is to find the narrowest common category. For example, in most business applications, clerks and managers would be best totaled as "employees," not "people."
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But note the dangers. "Employees" includes more than clerks and managers, and it slights the "people" aspect. No term captures what really goes on. The map is not the territory. If we miss this semantic aspect and things go awry, we may wrongly end up distrusting mathematics rather than our semantic habits. To make our numbers tell, we need to command both their math and their semantics.

And that's what I address in this book, how we should handle the combination of math and semantics. As you will discover, this combination includes more than enough critically important territory to be recognized as a "field" in its own right. I call this new field "mathsemantics." The word "mathsemantics," by the way, happily contains every letter in "mathematics" in the same sequence, plus just the letters s and n. Mathsemantics. Something New, slightly nutty, sounds nice.

Please be assured that this book is not a fussy theoretical treatise. My interests in numbers and semantics are deep but practical, based on my business needs and experience. I'm concerned about everyday failures to use figures sensibly, not abstract doctrinal questions. Although my subjects are math and semantics, I don't intend to sweat blood over definitions that don't matter. If given the choice, I'd rather say something approximately right about something important than say something precisely right about something unimportant. That's the approach I'll take here.

So, if you have picked up this book looking for a mathematical theory of meaning, you've been misled. My interest lies in the other direction, what we might call mathematical meanings in action or the semantics of numbers.

Sadly, traditional schooling mostly fails to solve the semantic problems involved in applying numbers to events. Educators say our schooling yields adults who fear to quantify or who fumble when they do attempt to quantify, adults apparently convinced that math is a powerful mystery they're not privileged to know. That schooling doesn't put math and semantics together. Indeed, semantics, if it appears at all, unfortunately ends up looking less like a key to understanding than a trivial fuss over words.
Well, you won’t need any special knowledge of math or semantics to read this book. If you followed the apples-oranges argument, you’re plenty intelligent—and open-minded—enough to grasp what I have to say.

Now, a promise. I promise you I’ve put no useless puzzles in this book, nothing designed just to show what marvelous things math can do, how smart I am (I’m not that smart), or how foolish people can look who don’t know the math tricks.

Puzzles and tricks tend to annoy slower but surer thinkers. I still remember my brother’s resentful complaint about geometry, a subject he reached three years before me. He said he didn’t like geometry; it was just a bunch of puzzles. His work that day at the blackboard had been criticized, he said, even though it contained no errors, because he’d taken more steps than Euclid had.

Well, this book won’t put you down for taking your own route. Quite the contrary. To “get” mathsemantics, I believe you need to think autonomously. The field of mathsemantics, as we’ll see, covers too much of life to be reduced to a bunch of fixed rules and procedures.

Mathematics does have power, elegance, mystery, and even great beauty. But this book concerns plainer stuff, like apples and oranges, using numbers more tellingly, and not making damn fools of ourselves.

Nevertheless, every now and again, mostly at the ends of chapters, I will summarize a main point in a form that respectfully salutes all mathematics and mathematicians. Thus, to generalize our apples-and-oranges finding,

**Proposition 1:** Whenever we add things, we necessarily add different things, which we must then group under the same name.
Perhaps the most consequential social rebuff I ever received was at the age of five. I approached three girls of my age and acquaintance and started to join them sitting on the grass. “Go away,” said one, “girls don’t play with boys.” Looking around, I could see the verdict was unanimous. So I acceded to their separatist demand, more or less for the next ten years. And for those ten critical years I was deprived. I lived beside girls, my sister was a girl. Yet my personal memories yield no insights into how it feels to be a girl from five to fifteen, how the world appears from that point of view.

Was it my pain at this sexist rebuff that led me ultimately to reject categories as rulers over what may or may not be brought together? I don’t know.

But anyway, before I came to delight in adding apples to oranges, I first had to suffer twelve years of normal schismatic instruction. At school, math was clearly one subject and English, the only place where semantic-like analyses seemed welcome, was clearly quite another. Like girls and boys, math and English existed side by side but in their separate realms, their distinctiveness more presumed than explained. Math was definite and precise, wasn’t it? And English less so, more feely-interpretive?
I can understand how its aura of precision entices us to exalt math. Like many people, I once believed that if accuracy and truth existed anywhere, mathematics must be the place. There was just no arguing a mathematical result. I remember my twelfth-grade report-card shock when my 92 in trigonometry plunged to a 63. Here was my first flunk since chorus in ninth grade when my voice went unruly. I summoned the courage to speak to our math teacher. “Mr. MacConnick,” I said, “you gave me a 63 in trigonometry.” “Yes, my boy,” he replied, “and you earned every point of it.”

Hah! No wonder I learned to use extreme caution with numbers. Who wouldn't fear such exacting authority?

Imagine this fear augmented in my virgin outing as an expert witness by the oath, “to tell the truth, the whole truth, and nothing but the truth.” I had to defend estimates I'd constructed to show Philadelphia’s need for an all-cargo transatlantic air carrier. The incumbent American-flag carriers, Pan Am and TWA, strongly opposed my estimates. Their attorneys had had several weeks to find holes in my exhibits. The cross-examination became pure torture. I did my novice best. I was cautious. Boy, was I cautious. I asked to have questions repeated. I looked for trouble and ambiguity in every word. I answered as carefully and narrowly as I could. I noted exceptions and stressed the sense in which I was answering. After two nerve-racking hours a recess was called.

Semantics then came to my rescue. Actually, it was a kindly judge, who put me wise to the meaning of my number-testimony. I've never mentioned it publicly before, for fear he may have broken some rule or other to help me out. “You needn’t,” he said quietly as he passed by, “be so circumspect.”

“Circumspect,” a magnificent word. “Circum,” around, as in circumnavigate or circumference. “Spect,” look, as in spectacle or inspect. Thus, “look around,” which has come to mean “careful to consider all circumstances and possible consequences,” not just “cautious.” How could a judge tell me not to be cautious? That's different.

My Merriam-Webster Collegiate dictionary puts the difference this way. “Cautious implies the exercise of forethought usually prompted
by fear of probable or even of merely possible danger; CIRCUMSPECT suggests less fear and stresses the surveying of all possible consequences before acting or deciding." What a judge! What an antidote to mathematical perfectionism! "You needn't be so circumspect."

Gradually in my career on the witness stand I came to accept that philosophical accuracy is a will-o'-the-wisp. What judges need is reasonable care—including usable numbers—and shorter hearings. Perhaps a hundred costly people that first day sat through my four hours on the stand. Lawyers said not to worry. Hadn't Pan Am and TWA staved off competition that much longer and wasn't everybody there getting paid?

I'm grateful it was a judge who tipped me off. A real authority figure, one to match even Mr. MacConnick. I'd taken my oath solemnly in swearing "to tell the truth, the whole truth, and nothing but the truth." Never mind that general semantics had already convinced me you can't possibly say all about anything. The formula still took me in. I tried to do the impossible.

I continued to take the oath seriously, but never after that day quite so literally. I consider myself honest and forthright, but I've never refused to take the oath. In Quaker school I'd learned of martyrs who'd refused on religious grounds to doff their hats in court. Even so, I've never presumed to confront a judge on the absurdity of swearing under penalty of perjury to do something impossible.

Arizona Pima County Superior Court Judge Lillian S. Fisher, writing in Newsweek about the rules of evidence, details case after case barring witnesses from telling the whole truth. In one, a remarried widow sues for damages resulting from her first husband's death. She testifies under her original married name. Mentioning remarriage or her new husband's support would result in a mistrial. So would mention of her first husband's life insurance policy, social security payments to her children, and payment of his medical bills by his health insurance.

She sues for damages as a woman deprived of her dead husband's affection and future life earnings. What the jury hears is neither exactly true (her name, for instance) nor is it the whole truth.

Judge Fisher allows that it wouldn't be right to change legal rules
just because they belie common sense. I agree. They might just have something else going for them. “But,” she concludes, “in the name of justice, we should at least change the oath that suggests that the witness will be telling ‘the truth, the whole truth, and nothing but the truth.’ ”

Mathematicians—well, at least many statisticians—dread the courtroom. I know this from their articles and editorials telling each other how to handle cross-examination and also from my own work preparing witnesses, including, yes, even some math teachers. I believe that most of them, like me that first day, try too hard to meet purely mathematical standards and too little to meet mathsemantic ones.

Facing students’ questions can give a teacher a false sense of authority and finality. In class, the teacher can be judge, jury, and executioner. But in court, the judge is in charge, your questioner is probably being paid to make you look foolish, and every word you utter goes into the record. Aaarrrrgh.

Still, there are things to like about courtrooms. Real issues get heard and settled, not for all time, but for the practical now. Even unpopular theories, if related to the issues, must be heard out and weighed.

I like the last most. My aviation analyses have sometimes worked better than my opponents’ because I know that a passenger isn’t a person but something a person does. Now, that’s unpopular, almost ungrammatical. Yet it’s good theory. It explains how one person can be several passengers a year, a useful mathsemantic truth. A passenger isn’t a person, but something a person from time to time chooses to be. An analyst losing sight of this temporal distinction risks turning any passenger study—no matter its mathematical refinements—into just so much high-sounding gobbledygook.

The challenge we face is not simply to understand math. That’s too abstract, too separatist, too elitist. No. The challenge is to understand ourselves, how we do or don’t put meanings into, and draw meanings from, numbers and mathematical relations. The challenge is personal, highly personal.

This challenge of personal interpretation affects you whether you’re
The challenge of togetherness

an atomic scientist, a housewife, or both, whether you’re a business manager or a civil-rights lawyer, a soldier or a sales rep. We all use numbers or have them used on us.

No matter how much math or semantic sophistication you happen to command, neither by itself is enough. To meet the mathsemantic challenge, you must muster some of both.

If we could talk face-to-face, it might be fun to address your specific interests and experiences, to compare them with mine. Given this book setting, however, the best I can do is try to make it easier for you to bridge the personal gap. For that, you need to know a few things about my background.

My primary area of expertness is air-service analysis. I’ve often been cross-examined on my use of various mathematical and statistical techniques. I am, however, definitely not a general expert in mathematics or statistics. My credentials are strong in general semantics and not bad in decision making. I’ve been published in all of these fields—air service, semantics, and decision making. I’ve worked in factories and office buildings, as blueprint trimmer, executive secretary, market researcher, company president, and management consultant. I know more than most people about administrative law, music, direct mail, and a few other things, but I’m not an expert in them.

The examples I use reflect my interests. You will, I hope, be able to link them to your interests. I’ve included many from recent newspaper and magazine reports. Others reflect my business experiences or the answers of applicants to our recruitment quiz. Some are intensely personal. Internalized math meanings are what this book is about. Hence the order of its presentation generally flows from more immediate personal concerns to longer-range social ones.

The examples in the next chapter ("Problems with names") sample current mathsemantic errors. The examples in the two succeeding chapters ("The magic of names" and "The magic undone") illustrate the root cause of the errors: childhood semantics embedded in language.

The next twelve chapters explore particular mathsemantic problems at the individual personal level. The remaining seven chapters present
a panorama of mathsemantic problems on a global interpersonal scale.

I hope you like to explore. I'm interested in all aspects of mathsemantics: the math side and the semantics side, childhood beliefs and stages, errors of all types, education and math anxiety, linguistic and cultural differences, evolution and history, math notation and number-memory, games and sports, childhood exercises that develop mathsemantic savvy, physical science and its philosophy, money and jobs, politics and the media, business and the professions, population and the environment, estimating and accounting, punctuality and time frames, gender differences, statistics and surveys, the future, what we can do about it, and so on; you name it.

How you approach this book obviously depends on you, whether you happen to be a word-person or a number-person or neither or both, your interests, your roles in life, and many other factors. I have no way of knowing, for example, whether you'd like to test your own mathsemantic savvy. If you would, you could tackle appendix A now or just answer each recruitment quiz question as it comes up. If, however you'd prefer to skip self-testing in favor of other aspects of mathsemantics, no problem; just read away. It's entirely up to you. I believe in personal autonomy.

I do, however, have two requests: First, please relax your semantic expectations. Try to subdue any feelings you might have that you need a concise, exact, complete, and compelling definition of "mathsemantics" up front. Just let your feel for the field develop naturally as we go along. This isn't math class, and "mathsemantics" isn't in the dictionary yet. Even if you could find it there, dictionary words about words never quite give the whole sense of a new term, anyway, do they? Let the rough definition of mathsemantics (the combination of math and semantics) and the apples-oranges example serve as enough for starters. I'll have more to say on this point at the end of the chapter after the next.

My second request is also semantic: Don't fret if you don't understand everything in this book. The March 1992 Notices of the American
Mathematical Society reported that math teachers, feeling they **had to** "get it" when math researchers spoke,

found it "enraging" that only five to ten minutes of the talks were comprehensible. Mathematicians commonly find themselves in this position during mathematics talks, but they usually don't worry about it.

So my second request amounts to asking you to read the way a mathematician would. Relax. **You don't have to get it all.**

Although mathematicians have written profound books on the foundations and meaning of mathematics, including such works as Whitehead and Russell's *Principia Mathematica* and Cassius Jackson Keyser's *Mathematics as a Culture Clue*, no one to my knowledge has written a whole book before on mathsemantics. The book you're reading is a first. It could give you an elegant, new, semantically oriented way of dealing with numbers and those who use numbers. It could even establish me as a new kind of expert.

Of course, on the down side, to put it delicately, it could also fail to establish the field. We'll see.

**Proposition 2:** Although some people may qualify as experts, no one can say the whole truth about anything.
I'd like to share with you a practical mathsematic problem I've had writing this book.

Our recruitment quiz posed five questions on percentages. None was answered correctly by even half the secretarial and clerical applicants. Please recall these applicants had represented themselves as "good at numbers" and passed our initial screening.

The analyst and administrative-law applicants, of course, did better. Most of them had taken some advanced math in school. Even so, their error rate on the five percentage questions varied from more than two out of five to just under one in five.

The questions were in the quiz because I'd noticed troubles with percentages. Nevertheless, before seeing the quiz results I hadn't realized the difficulties were this severe.

Apparently Paulos knew.

I'm convinced that a sizable minority of adult Americans wouldn't be able to pass a simple test on percentages, decimals, fractions, and conversions from one to another.
Our five questions required converting to percents from ordinary fractions, from decimals, and from a fraction expressed with a division sign. I believe they tested some of what Paulos had in mind. Given our results, my only question about his statement would be whether "minority" is correct.

My writing problem was how to communicate the quiz results. Expressing them in percents, the quiz evidence said, would lose readers. I considered but rejected the idea of giving an explanation of percentages up front. It would have had to precede the apples-and-oranges example in the first chapter and could well have annoyed both those who needed it and those who didn't.

Patricia Cohen notes an eighteenth-century preference for the form "one out of so many" where today we would use percents. That's exactly what I'd already done for seven chapters before I came across her statement. Call it an unconscious example of mathsemantic cultural perseveration.

If you're a devotee of percentages, you probably noticed my use of the "one out of so many" formula and wondered why I just didn't use percents. Now you know I was scared off by the quiz results. What would you have done in my place?

The five percentage questions in the quiz had the general direction, "Express the following items as percentages to the nearest whole percent."

The best results were obtained with translating common fractions into percents. The first of these was "7/10."

 Ninety-five of the one hundred ninety-six clerical and secretarial applicants, not quite half, gave the most frequent answer, which is also the correct one, "70%.

 The next most frequent answer, from forty-two applicants, about two in nine, was no answer. I take that to mean they knew they didn't know the answer and didn't want to guess.

 That leaves fifty-nine applicants, about three in ten. They gave fourteen other answers, each using one or more of just five symbols, 7, 1, 0, %, and . (decimal point). Of those using a percent sign, two answered "10%"; eighteen, "7%"; ten, "1%"; four ".7%"; one, ".70%";
and one, my favorite, simply “%.” Of those forgoing percent signs, five answered “70”; one, “10”; one, “7”; six, “1”; one, “1.0”; five “.7”; three “.70”; and one, “.07.”

The five who answered “70” might just have been too lazy to post a percent sign. The other fifty-four, however, were willing to hazard an answer but had only confused ideas of what “7/10” means, or what a percent is, or both. I wish I knew how they got their answers, but I don’t.

To anyone who loves words, “percent” explains itself. “Per” means “for each,” as in, for example, “one per person.” “Cent” comes from Latin “centum,” a hundred. It shows up in such words as “century,” “centennial,” and “centenarian,” all having to do with a hundred years, “centipede,” a hundred-legger, and “centimeter,” the hundredth part of a meter.

The best connection for understanding percents, however, is with “cent” as the hundredth part of a dollar. I’ve used this often in teaching employees. I tell them just to remember that one cent is also one percent of a dollar. Two cents is two percent of a dollar. Twenty-five cents is twenty-five percent of a dollar, and also a quarter (dollar). Fifty cents is fifty percent of a dollar, and also a half (dollar). A hundred cents is one hundred percent of a dollar.

Then I show employees how to find ten percent, one percent, and a tenth of one percent of any base number. Say the base number is 13,400.

\[
\begin{align*}
13,400 & = 100\% \text{, the base number itself} \\
1,340 & = 10\% \text{ of the base} \\
134 & = 1\% \text{ of the base} \\
13.4 & = 0.1\% \text{ of the base}
\end{align*}
\]

One can now divide any number by 13,400 and express the answer properly as a percent by locating the number’s interval in the table. I have employees make a similar table for another base and place a few numbers in the correct intervals. That’s usually enough instruction. The rest is practice.
Ninety-five applicants also converted the second common fraction, 
"3/5," correctly into "60 \%." Fifty gave no answer. The other fifty-one 
answers followed more or less the same pattern as for "7/10." The an-
wswers most extending the previous range were "3/5 \%" and "80 \%.”

Converting "\( .9 \)" gave the next best results. Eighty-four applicants, 
three in seven, gave the correct answer, "90 \%." Forty-six, two in nine, 
gave no answer.

Twenty-two, one in nine, answered "1 \%," an answer that could 
show a misunderstanding of decimals, or percentages, or both. Note 
that the money analogy would treat "\( .9 \)" as "\( .90 \)," ninety cents, ninety 
percent of a dollar, and therefore as ninety percent.

Eight applicants answered "9 \%," seven said plain "1," and four said 
"10 \%." The answers most extending the previous range were "\( .009 \)" 
and "130 \%.”

Converting "\( .814 \)" gave still poorer results. Sixty-eight applicants, 
just over one in three, got the correct answer, "81 \%." Fifty-two, more 
than one in four, didn't answer. Nineteen answered "80 \%," showing 
that rounding got in the way. The oddest answers were "\( 1/8 \% \)" and 
"1000 \%.”

Converting "\( 12 + 3 \)" produced the worst results. Only forty-nine 
applicants, one in four, gave the correct answer, "400 \%." Thirty-four 
didn't answer.

Forty-two said "4 \%,” thereby showing a misunderstanding of divi-
sion, or percentages, or both. Perhaps applicants didn't recognize 
"\( 12 + 3 \)" as another way of saying "\( 12/3 \).” I suspect, but can't prove, 
that some applicants had been drilled in converting fractions to per-
cents, but not fractions expressed this way. If so, they hadn't really un-
derstood what they were doing.

I won't describe in detail how the analyst and administrative-law 
applicants fared. All but five answered all five questions. The questions 
they answered best followed exactly the same sequence as the secre-
tarial and clerical applicants. Except for the appearance of equivoca-
tions, their wrong answers were like those already noted.

Ruedy says percents are “the most important part of practical
math, "dominating what you need in stores, for personal finances, and on the job. She says one can "learn percents once and for all in five minutes." On the mechanical level, that seems about right.

Her teaching method resembles ours. She explains that "percent" means "per hundred," makes use of the notion that cents are hundredths of dollars, shows how to figure what one and ten percent are, and provides some problems for quick drill.

Paulos says elementary schools teach fractions, decimals, and percents, but not how to convert from one to another.

Ruedy's book teaches conversions. She begins by stressing that "fractions, percentages, and decimals are different languages to express the same numbers." In six pages she then explains all six possible translations (fractions to percentages, fractions to decimals, percentages to decimals, and the reverse of each).

If we think of math as a language, perhaps we should treat fractions, percentages, and decimals as alternative formulations within that language, rather than as separate languages themselves. It's a mathsemantic question worth considering.

Ordinary languages like English are rich in alternative formulations. We use different voices, such as active ("The dog bit the man") and passive ("The man was bitten by the dog"). We use different moods, such as the indicative ("It's best to start by relaxing") and the imperative ("Please relax").

We translate easily between these formulations. For vigor, stylists advise converting sentences from the passive to the active voice. For clarity and brevity in the office, I phrase instructions in the imperative mood. None of this creates a new language.

If we think of fractions, decimals, and percentages as alternative formulations, we can retain the idea, which I prefer, that they're all part of one mathematical language.

Each formulation within a language presumably serves some purposes better than other formulations do. Otherwise it would disappear. This seems true of different voices and moods and also of fractions, decimals, and percentages.
It's easy to see that "1/2," "0.5," and "50%" all represent "half" and that they translate easily within this little set. Yet the three forms are not always so interchangeable.

Take a "third." Its fractional form is 1/3, which conveys the basic relationship of one to three as directly as possible. Everybody who's ever divided a candy bar three ways has literally felt the relationship.

The other forms usable for a "third" are less tangible. They create complications. The decimal form is a repeater, 0.33333..., as is the percentage form, 33.333...%. Repeaters require special symbolic conventions. Hand calculators employ inconsistent conventions and don't give simple fractional answers. In a letter to the editor printed in the New York Times on December 8, 1990, University of California mathematics professor M. H. Protter advanced these facts as an argument against allowing calculators in Scholastic Aptitude Tests.

Thus the decimal and percentage forms can create symbol-system problems the fractional form avoids. Therefore, fractions will in some circumstances be the preferred form. In other circumstances the preferred form will be decimals and in still others it will be percents.

Math instruction teaches the three forms. It may even show how to translate from one to the other, although many students apparently don't learn how very well.

We also need to ask when each form works best. Why do we usually state some things in fractions and others in decimals or percents? What are the advantages and disadvantages of each form? To what extent is it best to stick with one form and for how long? These questions go beyond math proper. They're mathsemantic questions.

Percents seem to work better than fractions and decimals for expressing comparative rates, changes in rates, and comparative changes in rates.

For example, if you have ten thousand dollars to invest, and can get six percent per year from one investment and seven percent from an equally safe one, then the higher rate will improve your position by one percent, one hundred dollars per year. You could say this in fractions or decimals, but it would be more roundabout, harder to grasp.

The advantage of percentages is even greater when we compare
Percentages

rates having different bases. For example, if a company that was profitable last year has this year increased its income (one base) by 13% and its costs (a separate base) by 9%, then we can say immediately that it's more profitable this year than last. Again, neither fractions nor decimals offer as easy an expression. As a consequence, we use percentages mostly to express rates destined to be compared, especially rates having different bases.

To understand a rate we need to know both what's being counted and the base to which it's being related. A report that housing starts are up three percent in March tells us neither. What are “housing starts”? How are they counted? Are they the actual housing starts in March or counts somehow adjusted for the weather or the season? What does “up three percent” mean? Up from what? Housing starts in February? Housing starts in March of last year?

Many news reports of percentage changes that I hear or read leave me wondering. I'm often unable to judge for myself whether the percent reported is good news or bad. Some reports are clearly wrong; others, just indeterminate.

The Philadelphia Inquirer of June 8, 1989, carried a story headlined, “Paramount bid sends Time Inc. stock up 40 pct.” The lead ran, “Time Inc.'s stock price yesterday soared more than 40 percent after Paramount Communications Inc.'s $10.7 billion buyout offer.” Further on the article stated, “Time yesterday jumped $44 to $170 a share.” The opening or base price, then, must have been $126. A $44 increase is about 35%. It looks like somebody mistook the $44 increase for a 44% increase. Of course, one can't be sure. Something else might be wrong.

Aviation Week of March 12, 1990, reported that the international routes of American Airlines “represent about 14% of its total system.” I don't know what this means, because I don't know what the base is. Is it route miles, passenger-miles, total passenger and freight revenues, or what? It's my field, I'm an expert, and still I don't know. Perhaps what makes me an expert is knowing that I don't know.

“Of the 64,000 square kilometers of Pamir territory,” began a story in the June 1990 issue of Soviet Life, “97.5 percent are mountain
ridges." That unnerved me. I thought ridges were lines along top edges, crests. A territory consisting almost entirely of edges struck me as a topological impossibility.

Wrong again. The dictionary brought me down nicely. One meaning of "ridge" is "a range of hills or mountains." Thank you, Merriam-Webster.

John Tesh on "Entertainment Tonight" reported that the PBS series "The Civil War" had an audience of 13% versus the usual 4%, "an increase of more than 300%." The percentage-point increase is 13% minus 4%, or 9%. The base is 4%. Therefore the percentage increase is not more than 300%, but only 9/4, or 225%.

Failure to allow for shifting bases leads to bad judgments. If sales drop by 20% and then grow by 25%, you’re now better off, right? Wrong. You’re back where you started. Proof: $100 minus 20% (one fifth) of $100 is $80. $80 plus 25% (one fourth) of $80 is $100. Add three zeros, or six, to each dollar figure for realism. Same conclusion.

Not all examples are so innocuous. A drop of 70% followed by a rebound of 80% leaves one in deep trouble. Proof: $100 minus 70% is $30. $30 plus 80% is only $54.

A gain of 80% followed by a loss of 70% is equally bad. $100 plus 80% is $180. $180 minus 70% is again $54.

I know. I know. It seems you can’t win. That’s not true. Many things fluctuate greatly and still gain. But in any given fluctuation, the percent going up has a smaller base than the percent coming down. Therefore, the percent gain must be larger than the percent loss, sometimes much larger, just to stay even.

If this shakes your faith in your ability to judge the effects of percentage changes, consider this: You may be better off for it. Now you know to work out the figures in dollars.

A Civil Aeronautics Board analyst had to recommend which airport, Dodge City or Garden City, Kansas, should be used if the cities consolidated their air service. The two airports offered similar patterns of convenience for their own residents and inconvenience for those of the
The predominant traffic flows from each city were east to Kansas City, Missouri, and beyond.

The analyst recommended use of Garden City, the more western Kansas airport. This would require most Dodge City passengers to backhaul before starting east. It defied common sense.

The analyst's reason? He'd calculated the local airport inconvenience as a percent of each airport's distance to Kansas City. This percent was lower for Garden City than for Dodge City. Therefore he favored consolidation at Garden City.

What he'd overlooked was that the distance to Kansas City was a variable base, larger from the more distant city. Because the local ground trips were equally inconvenient, dividing by the greater distance necessarily produced a lower percent. The analyst was a good guy. It was an unpleasant duty exposing the mistake.

Some people overlook bases so much they even add percents having independent bases. A. K. Dewdney, writing in the Scientific American of November 1990, reports a particularly flagrant example. A survey had shown that 49% of the Italian male and 21% of the Italian female respondents confessed to extramarital affairs. This led to the newspaper headline, “Seven Italians Out of Ten Have Committed Adultery.”

Not true. The percent for a combined base must lie between the percents for the separate bases. Thus we don't add 49% and 21%, but, assuming equal numbers of males and females, we take the average, 35%.

If the bases aren't equal, the combined percent will be closer to that with the larger base. Overlooking this leads to the famous horse-and-rabbit-stew error. A traveler was invited to have some horse-and-rabbit stew. “What,” he inquired, “are the proportions?” “Fifty-fifty,” he was told, “one horse, one rabbit.”

The eleven largest air carriers (the majors) and the next nine (the nationals) showed a combined traffic growth in July 1990 of 7.4% from the previous July. The traffic of the nine nationals alone increased by 23%. “Much of the growth,” concluded Aviation Daily, “was fueled by traffic surges among the national carriers.”
Not so. It's a horse-and-rabbit stew. The eleven major carriers combined carried about forty times the traffic of the nine nationals. The majors hadn't grown at as fast a rate, true, but they still contributed more than nine-tenths of the traffic increase.

The growth rates of some small companies typically exceed that of all larger companies, while the absolute growth of most larger companies exceeds that of all smaller companies.

<table>
<thead>
<tr>
<th>Company Size</th>
<th>Base Sales</th>
<th>Absolute Sales Growth</th>
<th>Sales Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>$100,000</td>
<td>$50,000</td>
<td>50%</td>
</tr>
<tr>
<td>Large</td>
<td>$10,000,000,000</td>
<td>$100,000,000</td>
<td>1%</td>
</tr>
</tbody>
</table>

Higher growth rates don't even necessarily mean that the smaller companies are catching up. In the example given above, the smaller company has just fallen further behind by $99,950,000.

Comprehension of rates, of one thing divided by another, is a considerable mathsemantic advance over comprehension of simple quantities. Recall the difficulty Piaget said children have understanding speed. They visualize it as the act of overtaking rather than as miles divided by time.

Conceptualizing comparative rates, changes in rates, and comparisons of changes in rates must be even more difficult than conceptualizing rates. Few people, apparently, acquire the knack. Errors abound.

Please plow through the following example.

The Aviation Daily of December 13, 1990, reported that “Eastern’s November 1990 traffic declined 5.3% from a year ago on 6.9% fewer available seat miles, causing the airline’s load factor to decline one percentage point.” This is impossible.

“Traffic” here means passenger-miles. (You'll recall the difficulty applicants had with multiplying unlike things. So we’re already off to a tough start.) Passenger-miles decreased by 5.3%. Available seat-miles (which I hyphenate) decreased by 6.9%. Therefore, passenger-miles decreased proportionately less than seat-miles.

“Load factor” means the percent that passenger-miles are of seat-
miles. If all seats were always filled, the load factor would be one hundred percent. If filled half the time, the load factor would be fifty percent.

So, if passenger-miles decreased proportionately less than seat-miles, then the load factor would, contrary to the story, go up, not down. Indeed, an accompanying table showed this is what actually happened.

If you can follow what I've just said, congratulations. If not, don't worry. The point I'm trying to make is that understanding how percents are used is not easy. Practically nobody receives the necessary training.

You may recall my mentioning that Ruedy said one could learn percents in five minutes and my agreeing that was about right for the mechanical level. As you can see, the pure math of percents may be easy, but their mathsemantic uses are hard.

Per cents in the abstract, in the pure math sense, are trivial, merely a change in the form of expressing a division. Five minutes is enough to learn them. Understanding the ways percents are used is definitely not trivial for living in today's world. Five minutes is definitely not enough. Practice with real per cents is needed.

But not just with baseball statistics. That's not good enough.

Philadelphians say that Rich Ashburn (also known as Whitey, and as Richie during his playing days) should be elected to the Hall of Fame. Among other things, he was the batting champion in 1955, hitting three thirty-eight, and again in 1958, when he hit three fifty. If you look these figures up, perhaps in the World Almanac, you'll find them in a column unambiguously headed "Pct.," but then reported as ".338" and ".350" in decimal form. On occasion someone will say, "One year Ashburn hit three hundred and fifty percent." Just think. Rich Ashburn hit three hundred and fifty percent! and he's not in Baseball's Hall of Fame.

Perhaps what we need is a name for "per thousand." Baseball isn't the only industry using "per thousands." Direct mail, for example, quotes mailing costs that way. What word could we use? Well, just as a "cent" is a hundredth part of a dollar, a "mill" is a thousandth part.
A millipede is a thousand-legger. So how about “permillage”? It’s in the dictionary. “Rich Ashburn hit three hundred and fifty permill.” Or second thought, forget it.

I suspect some reporters, perhaps partly misled by baseball statistics, lack the mathsemantic sophistication necessary to judge the percents appearing in their stories. Many otherwise sharp copy editors seem to have the same difficulty. That leaves them and us more at the mercy of whoever issues the figures.

Dewdney shows just how great the risk may be.

Once one puts it all together, it is easy to see how the media can play up the numbers, an all-too-common abuse I call numerical inflation. Joseph Childers of Bryte, Calif., has passed along documented claims about the percentages of fatal traffic accidents having various causes: cocaine, 20 percent (a New York newspaper); marijuana, 25 percent (Drug Enforcement Administration); alcohol, 50 percent (California Highway Patrol); sleepiness, 35 percent (sleep researchers); speeding, 85 percent (National Transportation Safety Board); smoking, 50 percent (National Highway Safety Administration); suicide, 35 percent (suicide researchers); mechanical failure, 20 percent (New York State Department of Motor Vehicles).

The total is 320%. That seems to leave no room for bad judgment in driving accidents but plenty in our thinking about them.

The same kind of room seems available in our thinking about crime, prices, the environment, international finance, company performance, population growth, marketing claims, war, and just about everything making the headlines these days.

**Proposition 27:** Percentages are dangerous social and economic tools that appear easy only to math teachers and the inexperienced.